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AN INSIGHT INTO WIRE ROPE GEOMETRY†

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Abstract—Cartesian coordinate equations, which describe the helix geometry of wires within a rope, are presented. Through the application of differential geometry and the use of the engineering drawing development approach, problems associated with the three-dimensional helix geometry of wire rope can be solved, allowing analysis of the geometrical properties. The geometrical analysis presented in this paper applies to any rope with axisymmetric strands.

NOMENCLATURE

a_h	helical wire radius in millimetres
b	binormal vector
i, **j**, **k**	unit vector associated with global Cartesian coordinates
K	curvature of the centroidal axis of the wire per unit length
r	position vector of space curve
R_w	helical radius of wire in millimetres
R_s	helical radius of strand in millimetres
R_d	drum radius in millimetres
R_r	ring radius in millimetres
S_r	length of rope in millimetres
S_s	path length of strand in millimetres
S_w	path length of wire in millimetres
T	unit tangent vector of space curve
X, Y, Z	global (that is, Cartesian) coordinates of space curve
x, y, z	local coordinates system of space curve
$\dot{X}, \dot{Y}, \dot{Z}$; $\ddot{X}, \ddot{Y}, \ddot{Z}$; $\dddot{X}, \dddot{Y}, \dddot{Z}$	derivatives of Cartesian coordinates with respect to θ_w
xy	defined parameter
α	helix angle of wire in a strand in degrees
β	helix angle of strand in a rope in degrees
γ	helix angle of rope wound around a drum in degrees
γ^*	double helix angle in degrees
θ	angle of rotation in degrees
$d\theta$	differential angle of rotation in degrees
θ_w	wire rotational coordinate in degrees
θ_s	strand rotational coordinate in degrees
θ_d	drum rotational coordinate in degrees
τ	torsion of helical wire per unit length
ρ_k	radius of curvature of the centroidal axis of the wire
ρ_τ	radius of torsion of the centroidal axis of the wire

Subscript

SD	double helix
DS	drum single helix
RS	ring single helix
DD	drum double helix
RD	ring double helix
w	helical wire
s	helical strand
r	rope
D	drum
R	ring

Superscript

′	binormal direction
±	direction of wire rotational coordinate in a Lang's lay rope
T	transpose of a matrix.

1. INTRODUCTION

A wire rope is a complex geometrical structure made up of many individual wires. The construction of wire rope gives it flexibility and as such it is an ideal structure for the transfer of tensile load. Under normal operating conditions wires within a rope are subject to varying degrees of mechanical damage. This damage is closely associated with the geometrical properties of the individual wires with which a rope is constructed. The degree of damage depends upon the geometrical and spatial configuration of the wires as well as their positions within the rope. A thorough understanding of the geometrical properties is required in order to model the deformation and strain components along individual wires within a rope under operating conditions.

1.1. *Geometrical construction of a stranded wire rope*

The construction of two types of stranded wire rope, namely six-strand and multi-strand, is described in this section to acquaint the reader with wire rope and construction terminology. The geometrical analysis presented in this paper, however, is not restricted to these types but applies to any rope with axisymmetric strands.

1.1.1. *Six-strand rope.* Wires which are wound around a central straight "king" wire produce a straight strand [Fig. 1(a)(i)]. The outer wires in this strand are all single helices. If now, for example, six of these strands are then wound around a central straight strand these outer strands will also have a single helical form. Similarly, the central core wire in each of these strands has a single helical form; however, the remaining wires in these outer strands each take on the form of a double helix. Such a structure is termed a six-strand

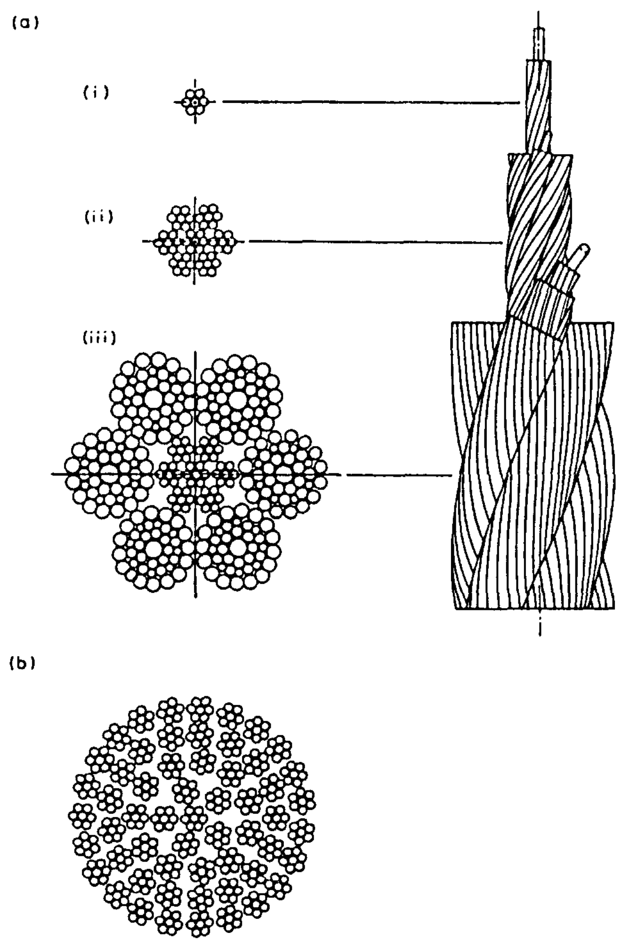

Fig. 1. Rope construction. (a) Six-strand rope. (i) Strand. (ii) Six-strand rope with a main core strand. (iii) Six-strand rope with an IWRC. (b) Multi-strand rope.

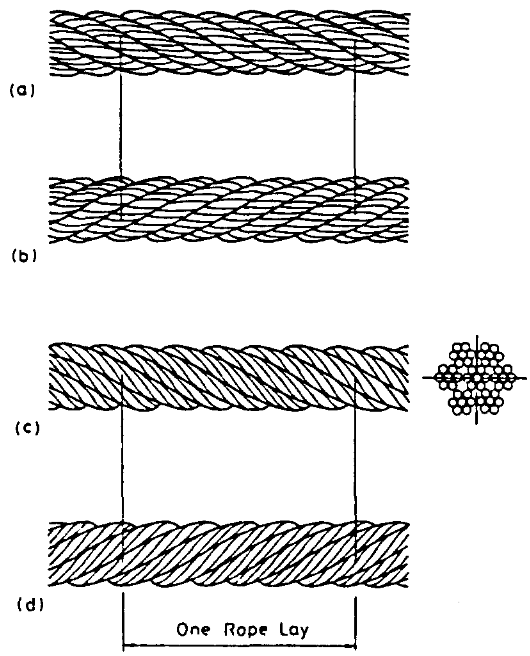

Fig. 2. Type, direction and length of lay. (a) Right-hand ordinary. (b) Left-hand ordinary. (c) Right-hand Lang's. (d) Left-hand Lang's.

rope (with a main core strand) [Fig. 1(a)(ii)]. This six-strand rope is termed an independent wire rope core (IWRC) if further strands are wound around it as shown in Fig. 1(a)(iii).

1.1.2. *Multi-strand rope.* If a straight strand has several layers of strands wound around it, a multi-strand rope is produced [Fig. 1(b)]. Whilst the central core strand is straight containing one straight "king" wire and (in this case) six single helical wires, the outer strands are in single helical form each one containing one single helical core wire and six double helical wires.

1.1.3. *Types and length of lay.* Where there are several layers of wire in a strand, the wires of one layer can cross over those of an underlying layer a number of times in each metre length of rope. This configuration helps to bind the rope together. However, it also causes internal wear in the form of discrete contact points because the wires of different layers cross over one another with certain contact angles. Strands laid in this way are referred to as being in cross lay. This internal wear can be reduced by adopting equal lay where the adjacent layers are effectively parallel to one another. If rope strands travel around in a clockwise direction (that is, in the direction of tightening a right-hand thread screw) the rope is in right-hand lay and if they run in the opposite direction the rope is in left-hand lay. When wires lie in the same direction as the strands lie in a rope the rope is in Lang's lay and when they lie in the opposite direction to that of the strands the rope is in ordinary lay. Stranded rope can therefore be produced as various combinations of Lang's (left hand or right hand) or ordinary (left hand or right hand). The distance over which a strand makes one complete rotation is known as a lay length (Fig. 2).

2. PREVIOUS WORK

Until recently mathematical models used to study wire ropes have been relatively simple and almost entirely restricted to strands made up of single helical wires (for example, the recent work by Phillips and Costello, 1973, 1985; Raoof, 1983; Utting and Jones, 1987). Furthermore, although the geometry of wire cross-sections governs the spatial configuration of the wires and strands in a rope, this has not been adequately considered in previous work. However, work has been carried out on wire rope with complex cross-sections by Velinsky *et al.* (1984), and on strand cross-sections in stranded ropes by Phillips and Costello (1985).

A relatively small amount of work has been published on the geometry of single and double helices in ordinary lay rope. In many practical applications, ropes are passed around sheaves or wound around drums and wires which are double helices in a straight rope then become triple helices. Despite this, triple helix geometry has not previously been considered in mathematical models.

Starkey and Cress (1959) considered the contact stresses of wires in a simple six-strand rope. Both "parallel" and "cross cutting" of straight cylinders were used to model the combination of contact situations. However, they assumed that the curved wires could be approximated by straight cylinders at the contact point. Stein and Bert (1962) removed this restriction in their analysis of the problem. They then presented the coordinate equations for the ordinary lay double helix and the equation for the curvature of this helix. The paper by Stein and Bert was very brief: a detailed derivation of the equations was not given.

Karamchetty (1978) attempted a study of the geometry of double helical wires. However, his equations do not agree either with those of Stein and Bert or those presented in this paper. For example, it should be possible to obtain the equations for Lang's lay from the equations of ordinary lay simply by reversing the direction of the wire rotational coordinate. This is not so for the equations presented by Karamchetty. Indeed, Karamchetty's equations do not distinguish between Lang's lay and ordinary lay at all.

The papers by Wiek (1986) and Knapp (1988) dealt mainly with the calculation of the radius of curvature of a single helical wire bent over a sheave. Their work on double helical wires is restricted to the degenerate limiting case of a strand bent into a circular arc.

Lee *et al.* (1987) carried out a more comprehensive study into rope geometry. They considered, for example, radii of curvature and torsion for the constituent wires when strand or rope is bent around a sheave or wound around a drum.

3. ASSUMPTIONS

In order to obtain the results given in this paper, the following assumptions were made:

(a) Any section normal to the centroidal axis of a wire (that is, any transverse section) is circular both before and after being bent over a sheave or wound around a drum.
(b) The shape of the centroidal axis is regarded as the most important geometrical characteristic of a wire.
(c) The shape of the centroidal axis of a curved wire within a rope is a helix; it may be either in the form of a single helix, double helix, or triple helix.

4. DEFINITIONS OF GEOMETRICAL PARAMETERS

The geometrical nomenclature of wire rope used in this paper is provided in Fig. 3 to which reference should be made when reading the definitions (a)–(d) and (g).

(a) *Wire helical radius* (R_w)

The helical radius of wire wound around any cylindrical strand is defined as the perpendicular distance from the centroidal axis of the wire to the centroidal axis of the parent strand.

(b) *Strand helical radius* (R_s)

For a strand wound around any type of cylindrical core, the strand helical radius, R_s, is defined as the perpendicular distance from the centroidal axis of the straight king wire to the centroidal axis of the core wire of a helical strand.

(c) *Wire rotational coordinate* (θ_w)

For two nearby points on the centroidal axis of a wire the differential $d\theta_w$ of the rotational coordinate θ_w is given by the angle between the osculating planes at the two

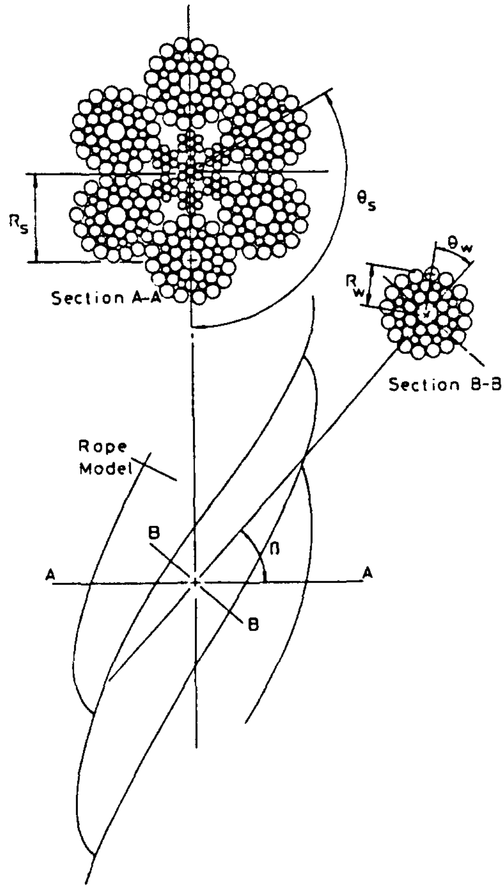

Fig. 3. Geometrical nomenclature of wire rope. Key: A–A transverse section of rope, B–B transverse section of strand, β helix angle of strand, R_s and R_w helical radius, θ_s and θ_w rotational coordinate.

points. The osculating plane at a point is defined as the plane formed by the tangential and normal vectors at that point.

(d) *Strand rotational coordinate* (θ_s)

For two nearby points on the centroidal axis of a strand wound around a core strand, an IWRC, or a multi-strand rope, the differential $d\theta_s$ of the rotational coordinate θ_s is defined as the angle between the osculating planes at the two points.

(e) *Ring/drum radius* (R_r *and* R_d)

If a strand or rope is passed over a sheave then the ring radius, R_r, is defined as the perpendicular distance from the centre line of the sheave to the centroidal axis of the strand or rope. Similarly, the drum radius, R_d, is defined as the perpendicular distance from the centre line of a drum to the centroidal axis of the strand or rope wound around the drum (Fig. 7).

(f) *Ring/drum rotation* (θ_r *and* θ_d)

For two nearby points on the centroidal axis of a strand or rope passed over a sheave the differential $d\theta_r$, of the ring rotation coordinate θ_r, is defined as the angle between the osculating planes at the two points. The drum rotation coordinate, θ_d, for a strand or rope wound around a drum can similarly be defined (Fig. 7).

(g) *Helix angle* (α, β, γ *and* γ^*)

The helix angle at any point along the centroidal axis of a wire in a rope is defined as the angle between the tangent vector to the axis and the plane normal to the axis. The helix angle for points along the axis of a strand or a rope is defined similarly.

For a strand in a straight rope the helix angle is a constant. The strand helix angle of a straight strand is 90°. It will be shown in this paper that for a wire with the order of its helical axis greater than one (that is, $n > 1$) the helix angle is a periodic function of position.

Definition of Helices

(i) *Single helix* (Borowski and Borwein, 1989). A curve with parametric equations

$$x = a\cos\theta \qquad y = b\sin\theta \qquad z = c\theta \tag{1a}$$

is a single helix whose axis is the Z-axis. For a circular helix the constants a and b are equal. The constant c determines the pitch (that is, lay length) of the helix.

(ii) *Double helix*. A double helix is a helical curve whose axis is a single helix. For example, wires wound around a single helical strand or a single helical strand wound around a drum. The parametric equations for a double helix are given in eqns (11) and (15).

(iii) *Triple helix*. A triple helix is a helical curve whose axis is a double helix. For example, a wire wound around a helical strand which is itself wound around a drum. The parametric equations for a triple helix are given later in this paper [eqns (17) and in the Appendix].

Remark. A helix can be a single helix, double helix, triple helix or even higher order. An nth order helix has a helical axis of $(n-1)$th order. A circle or a straight line can be considered as a degenerate limiting case of a single helix as the helix angle approaches 0° or 90° respectively. For a single helix, n is 1.

5. GEOMETRICAL MODELLING OF WIRE ROPE

5.1. *Application of differential geometry*

The centroidal axis of any wire in a rope is a three-dimensional space curve. It is convenient to use a local coordinate system at each point on the centroidal axis defined by the tangential, principal normal, and binormal vectors at that point. This is referred to as the Frenet frame at that point (Fig. 4). The position vector of a point on the centroidal axis is given in global Cartesian coordinates by

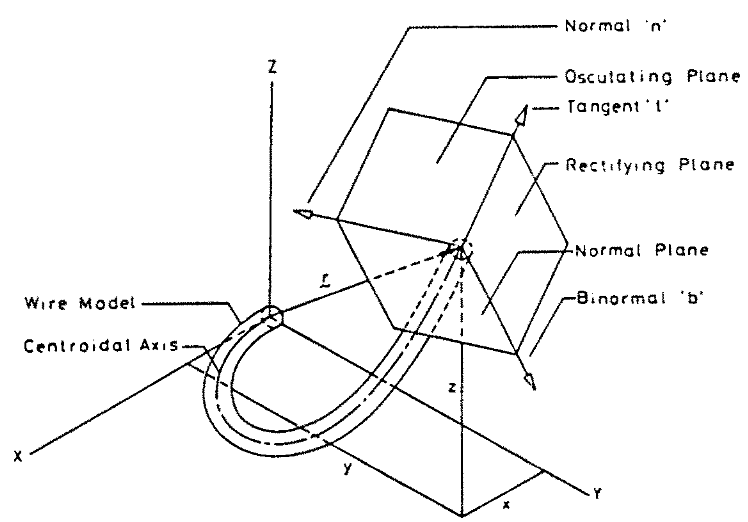

Fig. 4. Coordinate system of wire rope geometrical model. Key: **r** position vector, X–Y–Z global Cartesian coordinate, t–n–b Frenet frame.

$$\mathbf{r} = X\mathbf{i} + Y\mathbf{j} + Z\mathbf{k}. \tag{1}$$

The derivative of this, with respect to the variable parameterizing the curve, is

$$\dot{\mathbf{r}} = \dot{X}\mathbf{i} + \dot{Y}\mathbf{j} + \dot{Z}\mathbf{k}. \tag{2}$$

If the curve is parameterized by the angle of rotation θ_w, the distance dS between two nearby points on the curve is given by

$$\mathrm{d}S = |\dot{\mathbf{r}}|\,\mathrm{d}\theta_w, \tag{3}$$

that is,

$$\mathrm{d}S = (\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2)^{1/2}\,\mathrm{d}\theta_w. \tag{4}$$

The arc length between two points, $\theta_w = a$ and $\theta_w = \mathrm{b}$, is given by

$$S = \int_a^b |\dot{\mathbf{r}}|\,\mathrm{d}\theta_w. \tag{5}$$

Several expressions which are useful in calculating the geometrical properties of space curves are given below (see, as general references, Angus, 1975; Francis, 1978; Spiegel, 1981):

Curvature of a space curve

$$K = \frac{\{(\dot{Y}\ddot{Z} - \ddot{Y}\dot{Z})^2 + (\dot{Z}\ddot{X} - \ddot{Z}\dot{X})^2 + (\dot{X}\ddot{Y} - \ddot{X}\dot{Y})^2\}^{1/2}}{(\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2)^{3/2}} \tag{6}$$

Torsion of a space curve

$$\tau = \frac{\begin{vmatrix} \dot{X} & \dot{Y} & \dot{Z} \\ \ddot{X} & \ddot{Y} & \ddot{Z} \\ \dddot{X} & \dddot{Y} & \dddot{Z} \end{vmatrix}}{(\dot{Y}\ddot{Z} - \ddot{Y}\dot{Z})^2 + (\dot{Z}\ddot{X} - \ddot{Z}\dot{X})^2 + (\dot{X}\ddot{Y} - \ddot{X}\dot{Y})^2}. \tag{7}$$

Lee (1989) has presented the following expression for the helix angle [see Fig. 5(a) and (c)]:

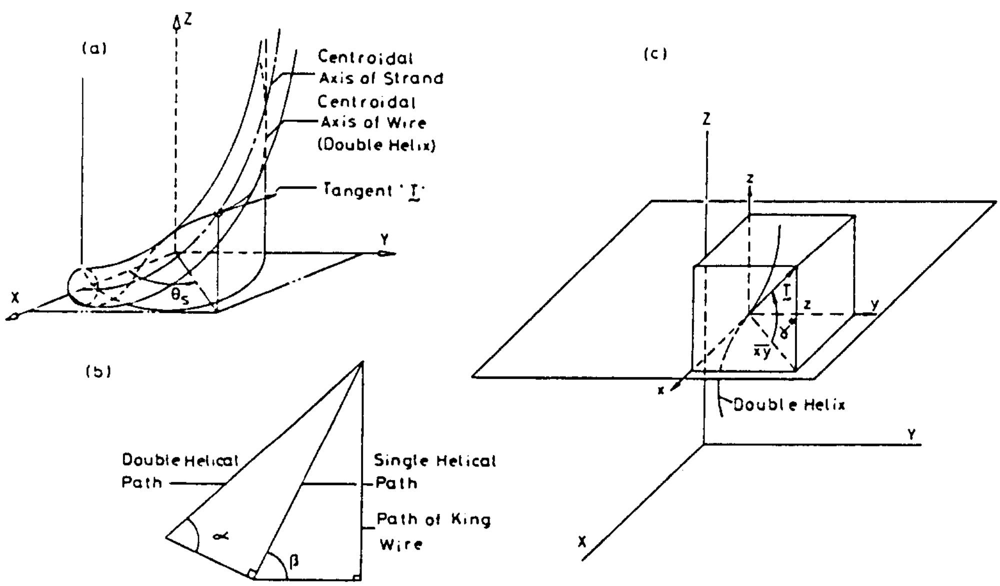

Fig. 5. Geometrical feature of double helix. (a) Rope model. (b) Development of double helical path. (c) Helix angle. Key: T tangent vector, γ^* double helix angle.

$$\gamma^* = \tan^{-1}\left\{\frac{\dot{Z}}{(\dot{X}^2+\dot{Y}^2)^{1/2}}\right\}. \tag{8}$$

5.2. *Development approach to geometrical analysis*

The engineering drawing development approach applied to rope helical geometry is based on the idea of projecting the centroidal axes onto a plane, without stretching or shrinking. It uses the fact that a cylinder is a developable surface (Mott, 1976; Abbot, 1987). The approach provides:

(a) a method for evaluating the path length of the centroidal axis of a strand or of a helical wire in a strand, and
(b) relationships between the wire, strand and rope rotational coordinates.

The developed path of a double helical wire in an undeformed rope is shown in Fig. 5(b). The expression for the path length can be obtained from Fig.5(b) by using simple trigonometry. Relationships for strands and ropes bent over sheaves or wound around drums can be obtained similarly, and are summerized in Table 1.

Another application of the development approach is to relate the different rotational coordinates in a rope (for example, θ_w and θ_s in a straight rope, Fig. 3). The rotational coordinates of helical wires and strands for a rope wound around a drum or bent over a sheave can be obtained in terms of the rotational coordinate of the drum or the sheave. Equations for double and triple helices can then be written in terms of any one of the rotational coordinates.

The relationship between the wire rotational coordinate, θ_w, and the strand rotational coordinate, θ_s, in an undeformed rope is

$$\theta_s = \frac{R_w}{R_s}\tan\alpha\cos\beta\theta_w. \tag{9}$$

The relationship between θ_s and θ_w for a strand wound around a drum is essentially the same, with γ, θ_d and R_d replacing β, θ_s and R_s respectively.

Table 1. Equations representing the path length of the centre-line of constituent wires, strand or rope using the development method

Path length (centre-line)	Expression
Straight single helical wire	$S_w = \frac{\theta_w R_w}{\cos\alpha}$
Straight double helical wire	$S_w = \frac{h_s}{\sin\alpha\cos\beta}; \quad h_s = R_s\theta_s$
(Alternatively)	$S_w = \left\{\left[\frac{R_s\theta_s}{\cos\beta}\right]^2 + R_w^2\theta_w^2\right\}^{1/2}$
Ring single helical wire	$S_w = (R_w^2\theta_w^2 + R_R^2\theta_R^2)^{1/2}$
Strand around a sheave	$S_s = R_R\theta_R$
Ring double helical wire	$S_w = (R_w^2\theta_w^2 + R_R^2\theta_R^2 + R_w^2\theta_w^2)^{1/2}$
Rope around a sheave	$S_R = R_R\theta_R$
Drum single helical wire	$S_w = \left[\theta_w^2 R_w^2 + \frac{R_D^2\theta_D^2}{\cos^2\gamma}\right]^{1/2}$
Strand around a drum	$S_s = \frac{R_D\theta_D}{\cos\gamma}$
Drum double helical wire	$S_w = \left[R_w^2\theta_w^2 + R_s^2\theta_s^2 + \frac{\theta_D^2 R_D^2}{\cos^2\gamma}\right]^{1/2}$
Rope around a drum	$R_R = \frac{R_D\theta_D}{\cos\gamma}$

If the centroidal axis of a wire in an undeformed rope is a double helix then, when the rope is wound around a drum, the axis will become a triple helix. The relationship between θ_w and θ_d is then

$$\theta_d = \frac{R_w}{R_d} \tan \alpha \sin \beta \cos \gamma \, \theta_w. \tag{10}$$

5.3. *Derivation of coordinate equations*

The shape of rope helices can vary considerably, depending on the location of the wires, the combination of helix angle and the lay directions of wires and strands within a rope. Also, the shape will depend on how the rope or strand is wound around a drum or bent over a sheave. The coordinate equations for a single helix are given in Angus (1975), Francis (1978) and Spiegel (1981); the coordinate equations for double and triple helices, together with some applications, are presented in this paper.

(a) *Double helix*. The geometrical properties of double helical wires in a rope depend on the helix angles and lay directions of the wires and strands in the rope. Double helical wires are found in the helical strands of a straight rope, in a strand bent over a sheave and in a helical strand wound around a drum.

In a double helix, the geometrical properties such as the helix angle, curvature and torsion vary cyclically. A rope is referred to as ordinary (or regular) lay if the orientation of the wire helix is opposite to the orientation of the strand helix; otherwise, it is referred to as Lang's lay. Equation (11) is derived by resolving the position vectors of points on a transverse strand section (section B–B in Fig. 6) onto the transverse rope section (section A–A in Fig. 6). The Cartesian coordinate equations, in matrix form, of the ordinary lay double helix can be written as:

$$\{\chi\} = \{O\}\{R\} \tag{11}$$

where

$$\{\chi\}^T = \{X, Y, Z\} \tag{12}$$

$$\{R\}^T = \{R_s, R_w, 0\} \tag{13}$$

and

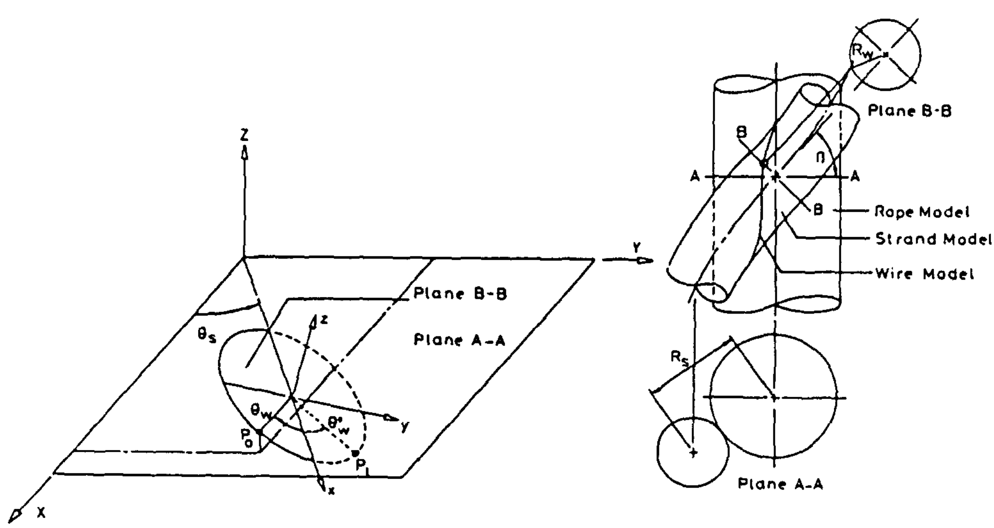

Fig. 6. Coordinate system of double helix (ordinary lay rope).

$$\{O\} = \begin{Bmatrix} \cos\theta_s & (\cos\theta_w\cos\theta_s + \sin\theta_w\sin\theta_s\sin\beta) & 0 \\ \sin\theta_s & (\cos\theta_w\sin\theta_s - \sin\theta_w\cos\theta_s\sin\beta) & 0 \\ \tan\beta\,\theta_s & \sin\theta_w\cos\beta & 0 \end{Bmatrix}. \tag{14}$$

The coordinate equations of the Lang's lay double helix can be obtained simply by reversing the direction of θ_w in eqn (14) (that is, replacing θ_w by $-\theta_w$) to give:

$$\{\chi\} = \{L\}\{R\} \tag{15}$$

where

$$\{L\} = \begin{Bmatrix} \cos\theta_s & (\cos\theta_w\cos\theta_s - \sin\theta_w\sin\theta_s\sin\beta) & 0 \\ \sin\theta_s & (\cos\theta_w\sin\theta_s + \sin\theta_w\cos\theta_s\sin\beta) & 0 \\ \tan\beta\,\theta_s & -\sin\theta_w\cos\beta & 0 \end{Bmatrix}. \tag{16}$$

(b) *Triple helix*. For a rope wound around a drum or bent over a sheave, the centroidal axis of the king wire forms a single helix, the centroidal axes of any single helical wires form double helices (refer to Fig. 7) and the centroidal axes of all of the double helical wires form triple helices. The geometrical properties of triple helical wires in a rope depend on the helix angles and lay directions of the wires, strands and rope.

The Cartesian coordinte equations for a triple helical wire, in a rope wound around a drum, are derived by considering three planes A–A, B–B and C–C. These are, respectively, the transverse planes of intersection of the drum, rope and strand. Figure 7 shows a double helical wire in a right-hand ordinary lay rope, wound around the drum in the right-hand direction. The coordinate equations are derived as follows. The position vectors (relative to the Frenet frame of the strand) of points in the wire section C–C are resolved onto the plane B–B. The position vectors of these mapped points, relative to the Frenet frame of the rope, are then resolved onto the plane A–A. This allows the geometry of the single helix to be used to calculate the triple helix coordinate equation (17).

The coordinate equations, in matrix form, for the triple helix can be written as:

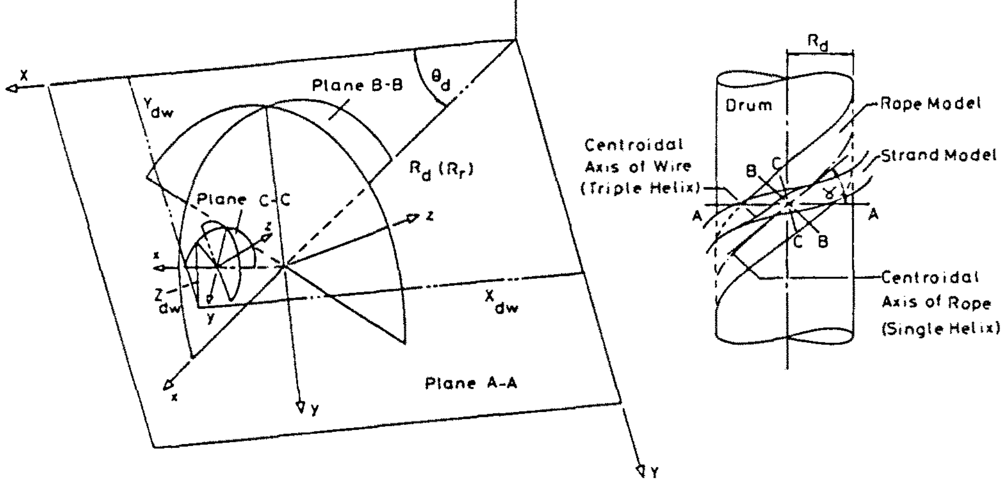

Fig. 7. Coordinate system of triple helix (ordinary lay rope). Key: γ helix angle of rope, A–A transverse section of drum, B–B transverse section of rope, C–C transverse section of strand.

$$\{\chi_{DD}\} = \{\chi_D\} + \{\chi_{sd}\} \tag{17}$$

with

$$\{\chi_{DD}\}^T = \{X_{DD}, Y_{DD}, Z_{DD}\} \tag{18}$$

$$\{\chi_D\}^T = \{X_D, Y_D, Z_D\} \tag{19}$$

$$\{\chi_{sd}\}^T = \{X_{sd}, Y_{sd}, Z_{sd}\} \tag{20}$$

where X_D, Y_D and Z_D are the coordinates of the centroidal axis of the rope. The expanded form of these equations is given in the Appendix to this paper.

5.4. *Calculating the geometrical properties of rope helices*

The coordinate equations for double and triple helices are given in terms of three parameters θ_w, θ_s and θ_d. However, θ_s and θ_d can be obtained from θ_w by using eqns (9) and (10), so the coordinate equations depend only on θ_w. The curvature and torsion of the helices can be obtained from the coordinate equations by differentiation.

6. DISCUSSION OF THE IMPLICATIONS OF ROPE GEOMETRY

The main results of the geometrical model for single, double and triple helices are now briefly presented and discussed. The relationship between the geometry of a rope and the type of damage to its constituent wires under cyclic loading has been discussed in more detail by Casey and Lee (1989).

The author has written a computer program to evaluate curvature, torsion, helix angles and other geometrical properties of rope helices. This program was used in drawing the graphs presented in this section.

6.1. *Single helix (reference should be made to Fig. 8)*

For a single helical wire the radius of curvature, radius of torsion and helix angle are constant along the length of the wire [Fig. 8(a)].

The curvature and torsion of a wire are related to the internal forces and moments by the equations of equilibrium presented by Love (1944). These equations imply that the internal forces and moments are constant along the length of each wire of a single layer and equal lay multi-layer straight strand subjected to monotonic tensile loading. Under dynamic loading this may not be the case because of non-linear effects, such as Coulomb damping (Nayfeh and Mook, 1979).

The helix angle of a single helical wire is usually between 60 and 90 ; within this range the radius of curvature, and to a lesser extent the radius of torsion, of the wire changes rapidly with helix angle [Fig. 8(b) and (c)]. Theoretically, the bending and torsional stress components along a large diameter single helical wire would be very sensitive to small changes in helix angle. Quantities such as the radial force, contact force and complementary shear force, which depend upon the bending and torsion, would also be very sensitive to changes in the helix angle. Bending and torsional stresses can be reduced by the use of smaller diameter wires. However, very small diameter wires (that is, with diameter less than 2 mm) can be susceptible to corrosion (National Coal Board 1980). Although bending and torsional stresses on a single helical wire surface can be reduced by using smaller diameter wire, the corresponding decrease in helical radius will, to some extent, affect these stresses level as the result of the decrease in radius of curvature and torsion [Fig. 8(d)].

6.2. *Double helix (reference should be made to Figs 9 and 10)*

Geometrical Properties: for a double helical wire the curvature, torsion and helix angle can be regarded as functions of θ_w. The mathematical model presented in this paper shows that:

(a) The curvature, torsion and helix angle are periodic functions of θ_w with a period of at most 360° [Figs 9(a), (b), (c) and 10(a), (b), (c)].
(b) For a Lang's lay rope the period of both curvature and torsion is 360°, with the two functions being 180° out-of-phase [Fig. 9(a) and (b)].
(c) For a strand wound around a drum (in ordinary lay), the torsion has a period of 360° but the period of the curvature may be less than this [Fig. 10(a) and (b)].
(d) The curve of the helix angle function will shift upward as the helix angle of the strand increases. For Lang's lay and ordinary lay ropes, both helix angle functions are 180° out-of-phase [Figs 9(c) and 10(c)].
(e) For a double helical wire in a Lang's lay rope, the minimum helix angle corresponds to the location where the maximum curvature and minimum torsion occur. For a double helical wire in an ordinary lay rope, the maximum helix angle corresponds to the location where the maximum curvature and minimum torsion occur (Fig. 9).

In order to visualize the geometrical implications of the double helical wires within a strand, the locations corresponding to different values of θ_w on the outermost layer of an outer strand are listed below:

(i) Points for which θ_w is a multiple of 360° are on the crown of the rope.
(ii) Points for which ($\theta_w - 180°$) is a multiple of 360° are points of contact with the strand layer immediately beneath the current strand layers.
(iii) Points for which ($\theta_w - 90°$) is a multiple of 180° are points of contact with neighbouring strands in the current strand layer.

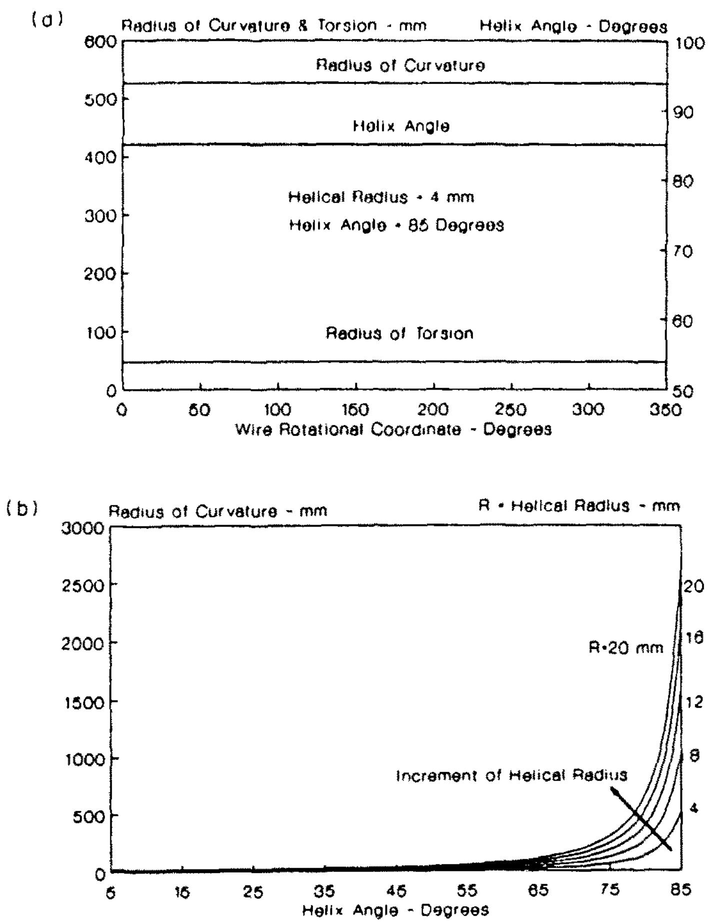

Fig. 8. (a) Geometrical properties of single helical wire. (b) Variation of the radius of curvature with helix angle. (c) Variation of the radius of torsion with helix angle. (d) Variation of the radius of curvature with helical radius.

(*Continued on next page*)

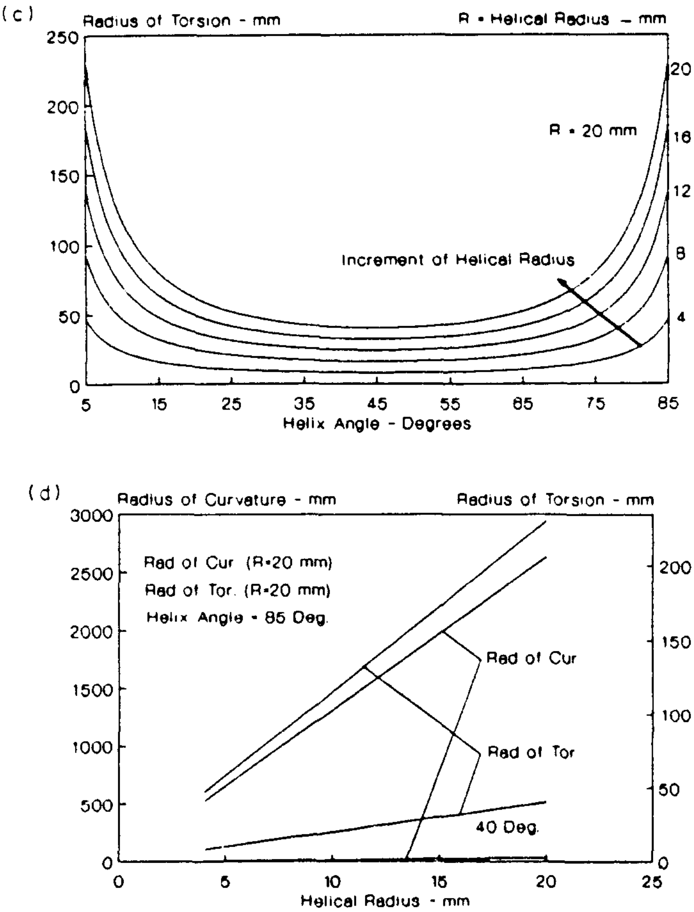

Fig. 8 — *continued*

Implications

Elastic rod theory shows that:

Bending moment = flexural bending stiffness × change in curvature.
Twisting moment = flexural twisting stiffness × change in torsion.

Combining these equations with the results of the geometrical model it can be seen that:

(a) Internal components of forces and moments will vary periodically with θ_w along a double helical wire, irrespective of the frictional condition imposed on the wire.
(b) If a rope is subjected to tension–tension fatigue tests the failure modes and the pattern of contact patches along a double helical wire will vary periodically with θ_w (see Casey and Lee, 1989).

For a rope which is not subjected to bending, the wire helix angle will in practice always be greater than 60°. Thus curvature will, to a good approximation, be 180° out-of-phase with torsion. This implies that, for a straight rope under tension, points of maximum bending will also be points of minimum twisting, and vice versa. Bending and twisting will be periodic in θ_w with a period of 360°.

The period of the curvature of a strand wound around a drum will be reduced if the drum helical radius is increased or the strand helix angle is reduced. For an ordinary lay rope the period of the curvature will also be reduced because the helix angle, for a strand wound around a drum, can be very small (less than 10°).

If a transverse section is made through the longitudinal axis of a rope the variation of the helix angle of a double helical wire is such that:

(a) The wire cross-section is approximately elliptical when the wire helix angle is a minimum and is circular when the wire helix angle is a maximum. The lay configuration of a rope can thus be identified from its transverse section.

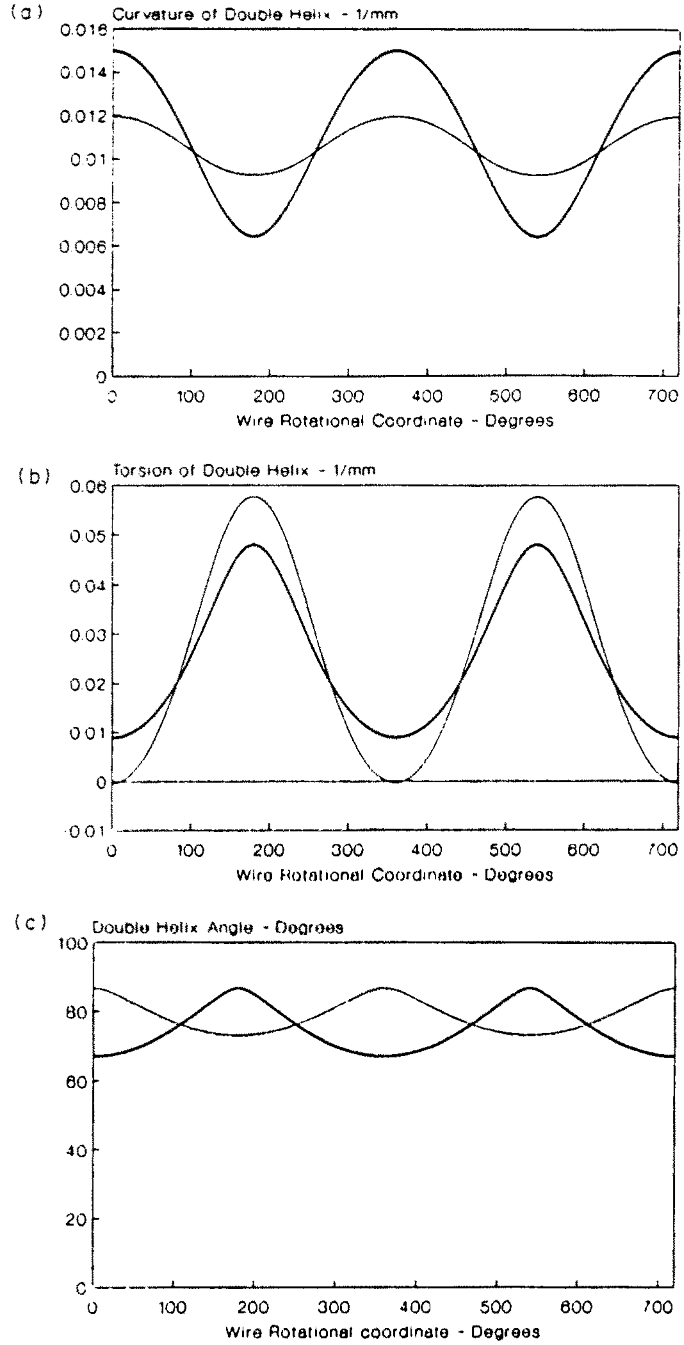

Fig. 9. Geometrical properties of straight double helical wire. (a) Variation of the curvature of a double helix with wire rotational coordinate. (b) Variation of the torsion of a double helix with wire rotational coordinate. (c) Variation of double helix angle with wire rotation coordinate. Key: helix angle = 80°, R_w = 2 mm, R_s = 6 mm, ---- Lang's lay, —— ordinary lay.

(b) When the wire helix angle is a minimum the curvature is a maximum and the torsion is a minimum. Similarly, when the wire helix angle is maximum the curvature is a minimum and the torsion is a maximum. These characteristics allow high bending and twisting stresses along a wire to be located.

From (a) and (b), it can be shown that if an ordinary lay rope with a Lang's lay IWRC is subjected to a tensile load, the maximum curvature and minimum torsion of a wire will occur in the regions of contact between the outer strands and the IWRC. The maximum torsion and minimum curvature occur at the crown of the outer strands.

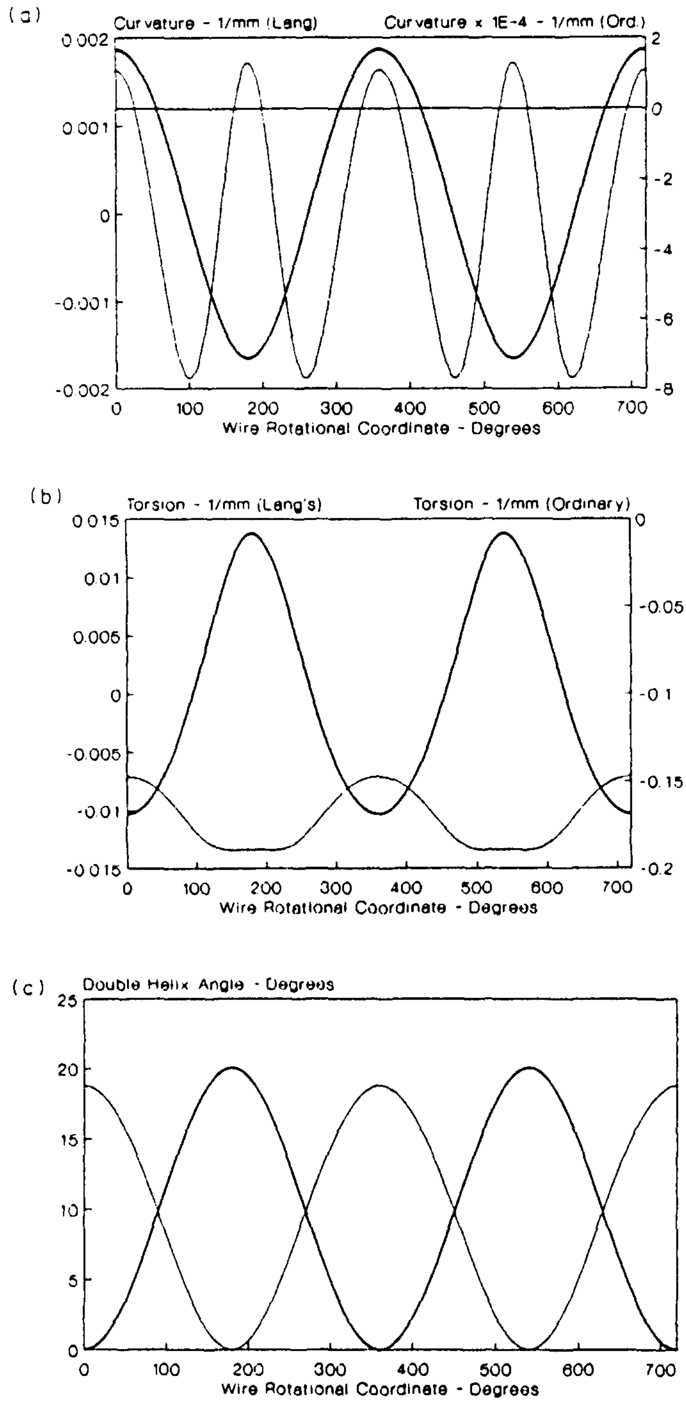

Fig. 10. Geometrical properties of drum single helical wire. (a) Variation of the curvature of a double helix with wire rotational coordinate. (b) Variation of the torsion of a double helix with wire rotational coordinate. (c) Variation of double helix angle with wire rotational coordinate. Key: R_w = 5 mm, R_s = 500 mm, —— Lang's lay, —— ordinary lay.

A single helical wire in a straight strand will be deformed into a double helical wire when the strand is wound around a drum. A single helical wire has a constant curvature; when the wire is deformed into a double helical wire the curvature will be a periodic function of θ_w which will lie above the θ_w axis. The graph of the difference between the double helix curvature and the single helix curvature will be the same shape as the graph of the double helix curvature but shifted downwards towards the θ_w axis. Similarly, the graph of the

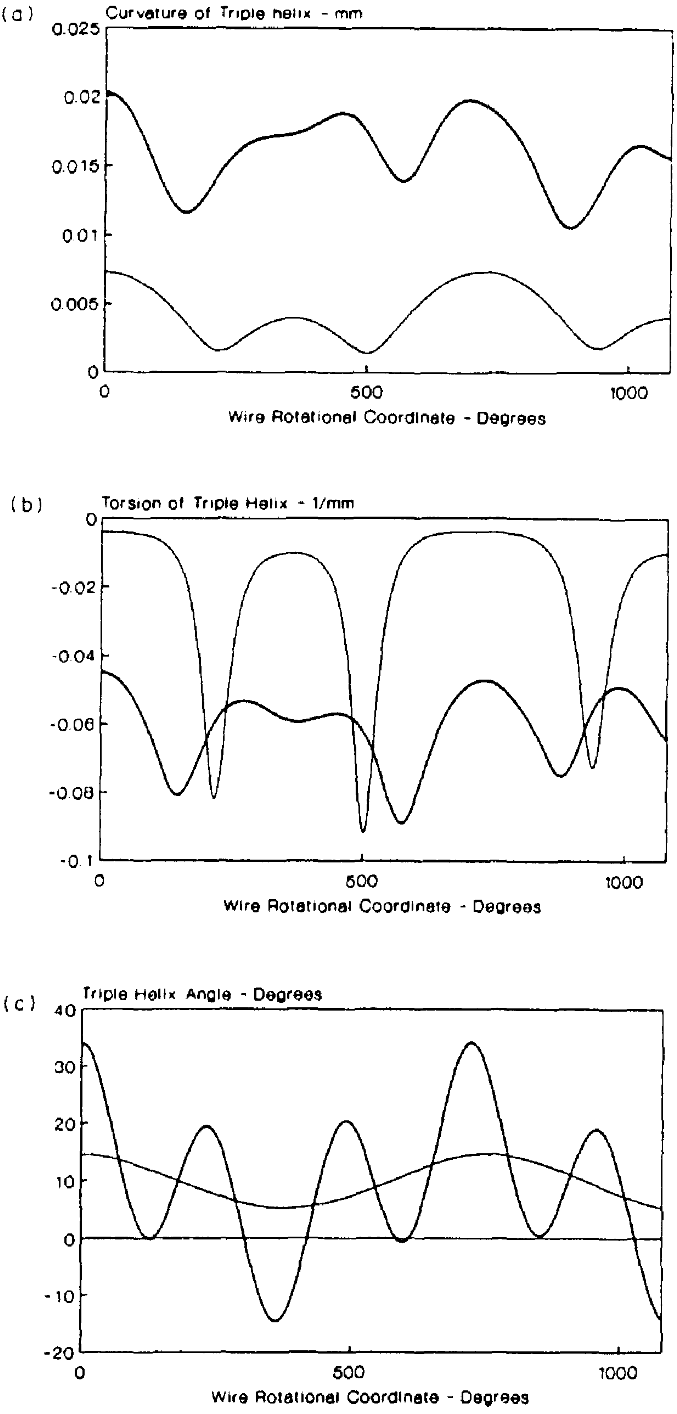

Fig. 11. Geometrical properties of drum double helical wire. (a) Variation of the curvature of a triple helix with wire rotational coordinate. (b) Variation of the torsion of a triple helix with wire rotational coordinate. (c) Variation of triple helix angle with wire rotational coordinate. Key: helix angle of wire = 80°, helix angle of strand = 80°, helix angle of rope = 10°, helical radius of wire = 4 mm, helical radius of strand = 8 mm, drum radius = 500 mm, —— Lang's lay, —— ordinary lay.

difference between the double helix torsion and single helix torsion will be the same shape as the graph of the double helix torsion, but shifted downward towards the θ_w axis.

The graphs of the curvature and torsion difference functions imply that:

(a) For a double helical wire in a Lang's lay strand the highest bending stresses will be at the crown on the top surface of the strand and the crown in contact with the drum.

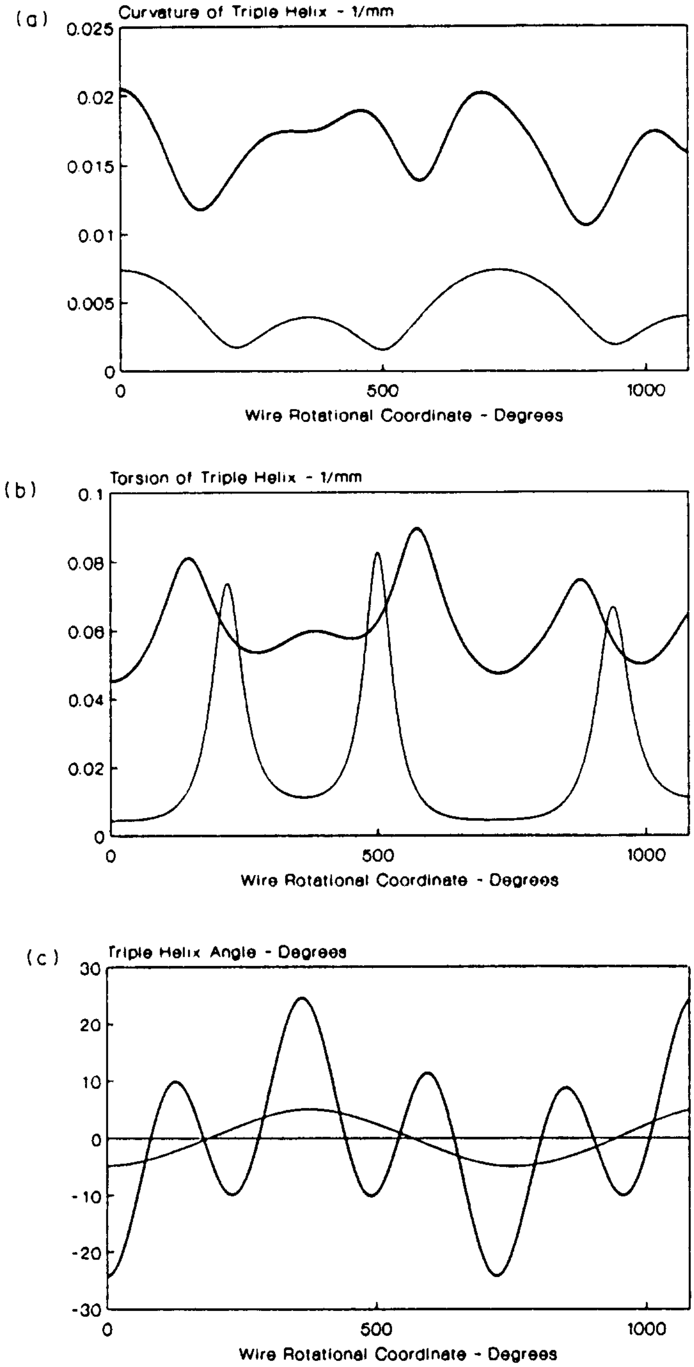

Fig. 12. Geometrical properties of drum double helical wire. (a) Variation of the curvature of a triple helix with wire rotational coordinate. (b) Variation of the torsion of a triple helix with wire rotational coordinate. (c) Variation of triple helix angle with wire rotational coordinate. Key: —— Lang's lay, —— ordinary lay.

(b) For a double helical wire in the outermost layer of an ordinary lay strand, the highest bending stresses will be at the region of contact with adjacent strands. At the crown, the magnitude of the twisting stress will be a maximum. The highest twisting stresses will be at the region of contact between the crown and the drum.

6.3. *Triple helical wire (reference should be made to Figs* 11 *and* 12)

The variations of curvature, torsion and helix angle for a triple helical wire are more complicated than for the double helical wire. The mathematical model presented in this paper shows that:

(i) The magnitude of the curvature of a triple helical wire in an ordinary lay rope is smaller than for a wire in a Lang's lay rope of the same size [Figs 11(a) and 12(a)]. In other words, bending stresses will be smaller in triple helical wires in an ordinary lay rope.
(ii) The variation in the torsion of triple helical wires in a Lang's lay rope is much less than in an ordinary lay rope [Figs 11(b) and 12(b)]; triple helical wires in a Lang's lay rope are therefore subjected to more twisting.
(iii) The variation of the helix angle for a triple helical wire in a Lang's lay rope is much greater than in an ordinary lay rope [Figs 11(c) and 12(c)].

The geometrical properties of a wire in a rope wound around a drum are determined by the direction of lay of the rope as it is wound around the drum, as well as the lay of the strands and wires in the undeformed rope.

For a rope wound around a drum, the mathematical model shows that the bending in double helical wires within a Lang's lay rope is greater than in double helical wires within an ordinary lay rope, for all values of θ_w. The torsion in a double helical wire in a Lang's lay rope is greater than in a double helical wire in an ordinary rope for most values of θ_w.

7. CONCLUSION

A mathematical model based on vector differential geometry and a development approach was used to investigate the geometrical properties of rope helices. A computer program derived from the mathematical model was used to calculate the geometrical parameters of double and triple helical wires in strands and ropes. The problems of strands and ropes bent around a sheave or wound around a drum were considered.

The wire curvature and torsion functions can be related to bending stresses. The properties of these functions and their implications for bending and twisting stresses depend on the lay of the rope; both ordinary lay and Lang's lay were discussed.

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APPENDIX: EXPANSION OF TRIPLE HELIX COORDINATE EQUATIONS IN MATRIX FORM

1. *Matrix* $\{\chi_D\}$

$$\{\chi_D\} = R_D\{I\}\{Q\} \tag{A1}$$

where

$\{I\}$ is a 3×3 unit matrix

$$\{Q\}^T = \{\tan\gamma\,\theta_D, \cos\theta_D, \sin\theta_D\}. \tag{A2}$$

2. *Matrix* $\{\chi_{sd}\}$

$$\{\chi_{sd}\} = \{A\}\{R\} \tag{A3}$$

where

$$\{R\}^T = \{R_s, R_w, 0\} \tag{A4}$$

$$\{A\} = \begin{Bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ A_{31} & A_{32} & 0 \end{Bmatrix}. \tag{A5}$$

Rope wound around drum in the left-hand direction

(a) The elements of matrix $\{A\}$ for a right-hand Lang's lay rope

$$\begin{aligned}
A_{11} &= \sin\theta_s\cos\gamma \\
A_{12} &= \cos\theta_w\sin\theta_s\cos\gamma + \sin\theta_w(\cos\theta_s\sin\beta\cos\gamma - \cos\beta\sin\gamma) \\
A_{21} &= \cos\theta_D\cos\theta_s + \sin\theta_D\sin\theta_s\sin\gamma \\
A_{22} &= [\cos\theta_w\sin\theta_s\sin\gamma + \sin\theta_w(\cos\theta_s\sin\beta\sin\gamma + \cos\beta\cos\gamma)]\sin\theta_D \\
&\quad + (\cos\theta_w\cos\theta_s - \sin\theta_w\sin\theta_s\sin\beta)\cos\theta_D \\
A_{31} &= \sin\theta_D\cos\theta_s - \cos\theta_D\sin\theta_s\sin\gamma \\
A_{32} &= -[\cos\theta_w\sin\theta_s\sin\gamma + \sin\theta_w(\cos\theta_s\sin\beta\sin\gamma + \cos\beta\cos\gamma)]\cos\theta_D \\
&\quad + (\cos\theta_w\cos\theta_s - \sin\theta_w\sin\theta_s\sin\beta)\sin\theta_D.
\end{aligned} \tag{A6}$$

(b) The elements of matrix $\{A\}$ for a right-hand ordinary lay can be obtained by reversing the direction of θ_s in eqn (A5)

$$\begin{aligned}
A_{11} &= -\sin\theta_s\cos\gamma \\
A_{12} &= -\cos\theta_w\sin\theta_s\cos\gamma + \sin\theta_w(\cos\theta_s\sin\beta\cos\gamma - \cos\beta\sin\gamma) \\
A_{21} &= \cos\theta_D\cos\theta_s - \sin\theta_D\sin\theta_s\sin\gamma \\
A_{22} &= [-\cos\theta_w\sin\theta_s\sin\gamma + \sin\theta_w(\cos\theta_s\sin\beta\sin\gamma + \cos\beta\cos\gamma)]\sin\theta_D \\
&\quad + (\cos\theta_w\cos\theta_s + \sin\theta_w\sin\theta_s\sin\beta)\cos\theta_D \\
A_{31} &= \sin\theta_D\cos\theta_s + \cos\theta_D\sin\theta_s\sin\gamma \\
A_{32} &= [\cos\theta_w\sin\theta_s\sin\gamma - \sin\theta_w(\cos\theta_s\sin\beta\sin\gamma + \cos\beta\cos\gamma)]\cos\theta_D \\
&\quad + (\cos\theta_w\cos\theta_s + \sin\theta_w\sin\theta_s\sin\beta)\sin\theta_D.
\end{aligned} \tag{A7}$$

Rope wound around drum in the right-hand direction

(a) The elements of matrix $\{A\}$ for a right-hand Lang's lay can be obtained by reversing the direction of θ_D in eqn (A5)

$$\begin{aligned}
A_{11} &= \sin\theta_s\cos\gamma \\
A_{12} &= \cos\theta_w\sin\theta_s\cos\gamma + \sin\theta_w(\cos\theta_s\sin\beta\cos\gamma - \cos\beta\sin\gamma) \\
A_{21} &= \cos\theta_D\cos\theta_s - \sin\theta_D\sin\theta_s\sin\gamma \\
A_{22} &= -[\cos\theta_w\sin\theta_s\sin\gamma + \sin\theta_w(\cos\theta_s\sin\beta\sin\gamma + \cos\beta\cos\gamma)]\sin\theta_D \\
&\quad + (\cos\theta_w\cos\theta_s - \sin\theta_w\sin\theta_s\sin\beta)\cos\theta_D \\
A_{31} &= -\sin\theta_D\cos\theta_s - \cos\theta_D\sin\theta_s\sin\gamma
\end{aligned}$$

$$A_{32} = -[\cos\theta_w \sin\theta_s \sin\gamma + \sin\theta_w (\cos\theta_s \sin\beta \sin\gamma + \cos\beta \cos\gamma)] \cos\theta_D$$
$$- (\cos\theta_w \cos\theta_s - \sin\theta_w \sin\theta_s \sin\beta) \sin\theta_D. \qquad (A8)$$

(b) The elements of matrix $[A]$ for a right-hand ordinary lay can be obtained by reversing the direction of θ_D and θ_s in eqn (A5)

$$A_{11} = -\sin\theta_s \cos\gamma$$
$$A_{12} = -\cos\theta_w \sin\theta_s \cos\gamma + \sin\theta_w (\cos\theta_s \sin\beta \cos\gamma - \cos\beta \sin\gamma)$$
$$A_{21} = \cos\theta_D \cos\theta_s + \sin\theta_D \sin\theta_s \sin\gamma$$
$$A_{22} = [\cos\theta_w \sin\theta_s \sin\gamma - \sin\theta_w (\cos\theta_s \sin\beta \sin\gamma + \cos\beta \cos\gamma)] \sin\theta_D$$
$$+ (\cos\theta_w \cos\theta_s + \sin\theta_w \sin\theta_s \sin\beta) \cos\theta_D$$
$$A_{31} = -\sin\theta_D \cos\theta_s + \cos\theta_D \sin\theta_s \sin\gamma$$
$$A_{32} = [\cos\theta_w \sin\theta_s \sin\gamma - \sin\theta_w (\cos\theta_s \sin\beta \sin\gamma + \cos\beta \cos\gamma)] \cos\theta_D$$
$$- (\cos\theta_w \cos\theta_s + \sin\theta_w \sin\theta_s \sin\beta) \sin\theta_D. \qquad (A9)$$